Differentiation Critical Points Story

This story is about what a critical point is and how they are found. It explores the definition and discovery of critical points using functions and graphs as well as possible uses for them in the everyday world.

Critical points are key in calculus to find maximum and minimum values of graphs. At this point you may be saying to yourself 'who cares,' or 'why do we need this?' You may be surprised.

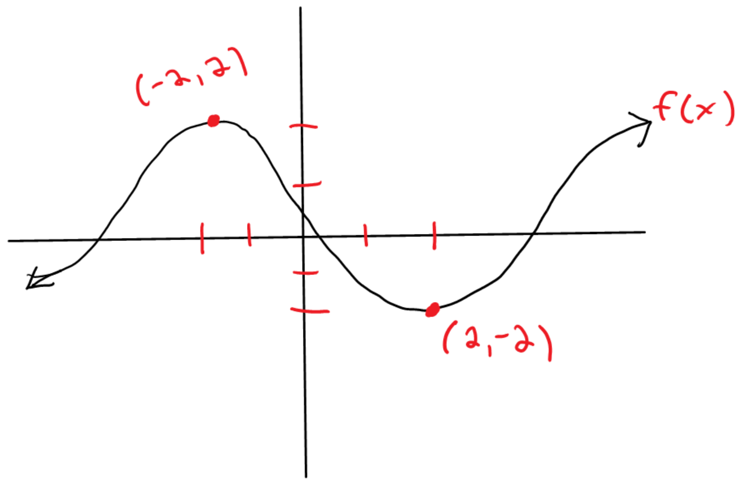
Let's say you bought a new dog and went down to the local hardware store and bought a brand new fence for your yard, but alas, it doesn't come assembled. Of course, this means that you get to fence in whatever size lot you want with restrictions of how much fence you have. Wouldn't you want to maximize the amount of space your dog had to run? Critical points can tell you the exact dimensions of your fenced-in yard that will give you the maximum area!

Critical points in calculus have other uses too. For example, they could tell you the lowest or highest point of a suspension bridge (assuming you can plot the bridge on a coordinate plain). Now we know what they can do?, but how do we find them? First, let's officially define what they are.

Let f be defined at b. If $f'(b) = 0$ or if f is not differentiable at b, then b is a **critical number** of f. If this critical number has a corresponding y value on the function f, then a critical point exists at (b,y).

What exactly does this mean? Well, f just represents some function and b represents the point or number that we are looking for. The second part of the definition tells us that we can set the derivative of our function equal to zero and solve for x to get the critical number! The third part says that critical numbers may also show up at values in which the derivative does not exist. We will see an example of this below. Lastly, if the critical number can be plugged back into the original function, the x and y values we get will be our critical points.

Let us look at a graph, point out some critical points and try to find why we set the derivative equal to zero.

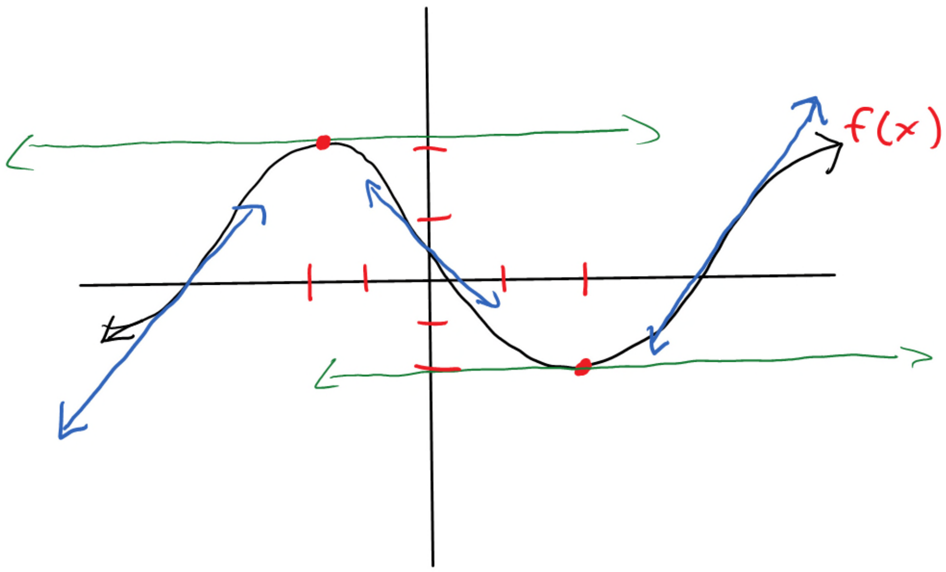


The red dots on the graph represent the critical points of that particular function, f(x). It's here where you should start asking yourself a few questions:

1. Is there something similar about the locations of both critical points? *You should look for visual similarities.*
2. How does this compare to the definition from above?

If you understand the answers to these 2 questions than you can understand how we find critical points.

1. Notice how both critical points tend to appear on a hump or curve of the graph. More specifically they are located at the very top or bottom of these humps. Mathematically speaking, the slope changes from positive to negative (or vice versa) at these points. It's why they are so critical!
2. To understand how number one relates to the deflection of a critical point, we have to remember what exactly a derivative tells us. The derivative of a function, *f(x)*, gives us a new function *f'(x)* that represents the slopes of the tangent lines at every specific point in *f(x)*. So why do we set those derivatives equal to zero to find critical points? Take a look at the following graph that shows different tangent lines to f(x).

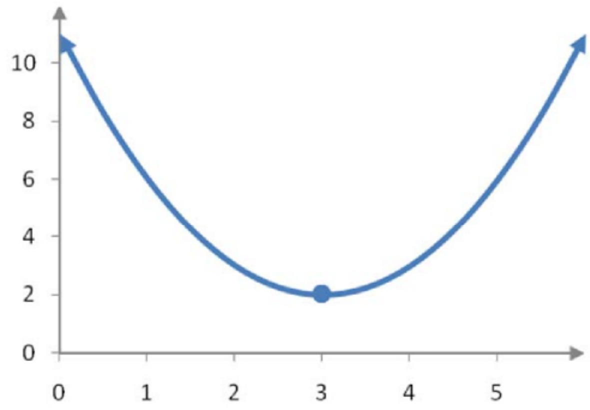


The Green Tangent Lines run through our critical points. What's the difference between those and the blue ones? For one thing, they have the same slope whereas the blue tangent lines all have different slopes. For another thing, that slope is always one very specific number (who remembers the slope of a horizontal line?).

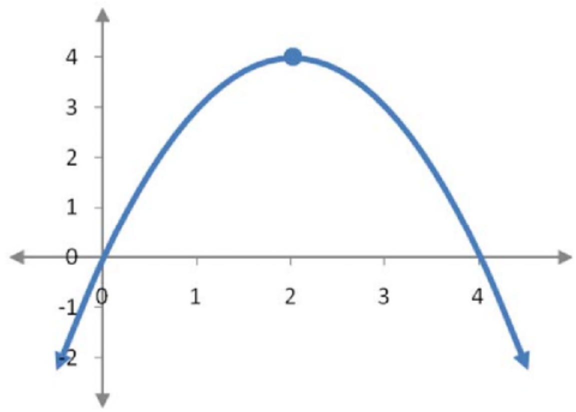
That's right! The slope of every tangent line that passes through a critical point is always **zero**!

Let us look at another illustration.

The graph of a function may rise and fall resulting in relative and absolute extrema. For instance, if a continuous function falls and then rises as we move from left to right on its graph then we can deduce that the function must have a relative minimum.



a



b

When a function falls and then rises, we get a relative minimum as in (a). In (b), the function rises and then falls resulting in a relative maximum. A continuous function that rises on the left and then drops on the right has a relative maximum. The terms increasing and decreasing are used to describe when a function is rising or falling as we move from the left to the right on the graph of a function.

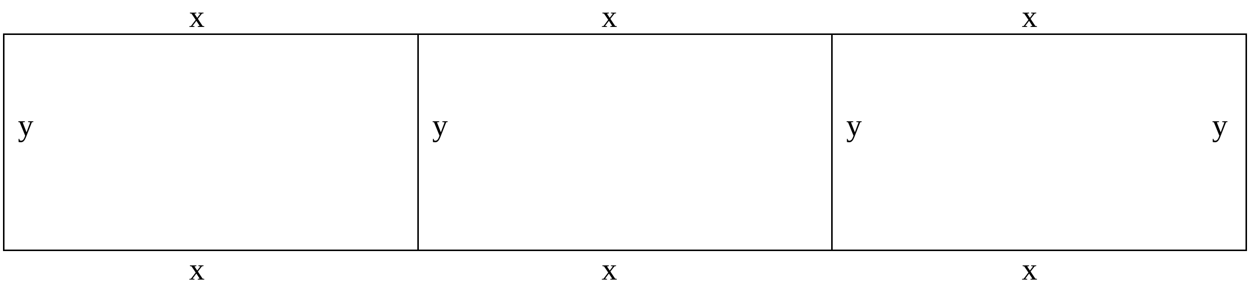
Suppose x_1 and x_2 are two numbers in an interval over which a function f is defined.
f is increasing on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$. f is decreasing on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.

Critical points are ordered pairs on the graph where the function can potentially change from increasing to decreasing or decreasing to increasing. This means that we will be able to use these critical numbers to find the relative extrema of a function.

Let f be a continuous function defined on an open interval containing a number c. The number c is critical value (or critical number) if $f'(c) = 0$ or $f'(c)$ is undefined. A critical point on the graph of f has the form $(c, f(c))$.

A Practical Example

A rancher has 400 feet of fencing and wants to build a corral that is divided into three equal rectangles as shown below:

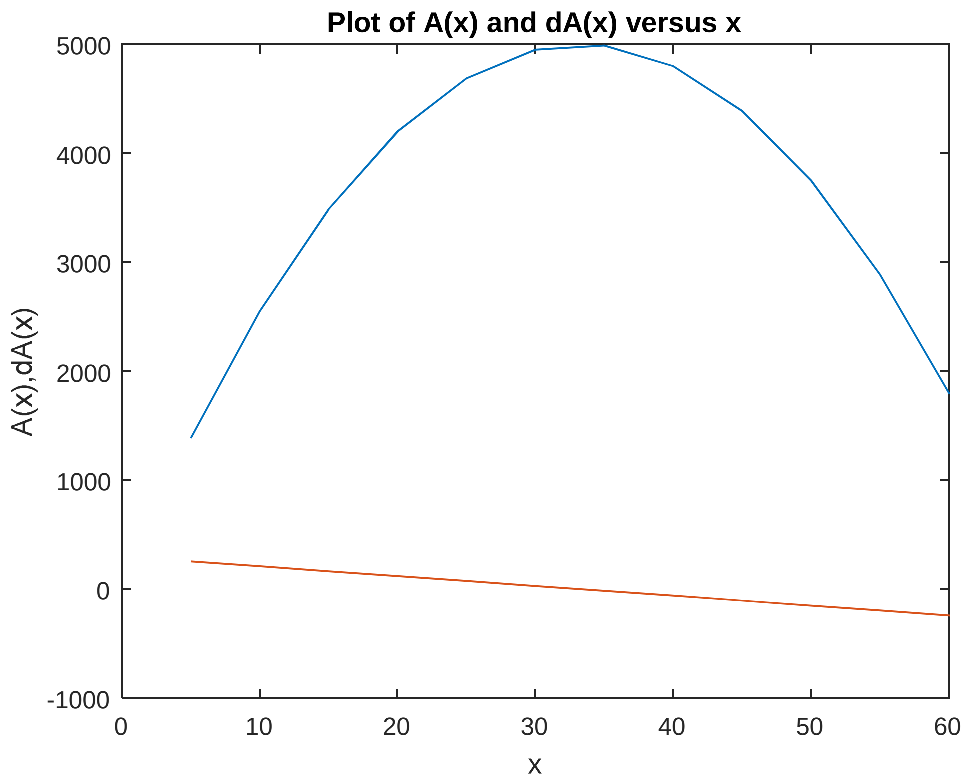


Corral area:

A=length*width
A(x,y)=3*x*y
Now,
6x+4y=400, or
3x+2y=200, Therefore, y=100-1.5x (1)
Substituting above
A(x,y)=3*x*y
A(x)=3x*(100-1.5x)
A(x)=300x-4.5x^2 (2)
Differential A'(x)=300-9x
Equating the differential to zero, the critical point
300-9x=0
Therefore, 9x=300, or
x=100/3
Substituting in (2)
A(100/3)=300*100/3 -4.5*$(100/3)^{2=}$10000-4.5*10000/9=10000-5000=5000
Therefore,
A(100/3)=5000, for x=100/3 and from (1) y=100-1.5*100/3=50
Answer
The corral width=3*x=100 and depth=50 for total area of 5,000

In MATLAB (using syms – symbolic manipulation tool) :

```
>> syms x
>> A(x)=300*x-4.5*x^2;
>> dA=diff(A);
>> x=5:5:60;
>> plot(x,A(x))
>> hold on
>> plot(x,dA(x))
>> xlabel('x')
>> ylabel('A(x),dA(x)')
>> title('Plot of A(x) and dA(x) versus x')
```



Answer,

Reference Link:

http://study.com/academy/lesson/finding-critical-points-in-calculus-function-graph-quiz.html

http://pblpathways.com/calc/C12_1_2.pdf

Ryan, Mark, Calculus Workbook, 2e, Wiley, 2015, pp.152